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NASA TN D-93

NASA TN D-93

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TECHNICAL NOTE
D-93

APPLICATION TO FLUID DYNAMICS OF THE THEORY

OF REVERSIBLE HEAT ADDITION

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON October 1959

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

A discussion is given of the approximations required to relate the equations of Hicks and Shapiro for gas flows with heat addition to the general relations for radiating, reacting gas flows given by Hirschfelder, Curtiss, and Bird. It is indicated that the theory of Hicks and Shapiro corresponds to reversible heat addition because of the absence of dissipative terms in the momentum equations. When applied to combustion problems, the theory is applicable to air-breathing engines at speeds up to about half of satellite speed. The theory of reversible heating is applied to underwing heat addition at hypersonic speeds, and the results of a series of exact calculations are given. The effects on aircraft range of several arrangements of external heat addition are presented.

INTRODUCTION

Three types of flow which are of practical interest in aeronautics, but for which a relatively small number of solutions are available are (1) those associated with combustion problems, (2) flow of a gas not in equilibrium with respect to all degrees of freedom, and (3) flow of a gas under conditions such that the flow field is partially determined by radiation absorbed and emitted by the gas.

In this work, a self-consistent theory of inviscid flow with reversible heat addition has been selected for study. The applicability of this theory to the types of flow listed above will be discussed briefly. Methods for obtaining solutions will be described and several solutions applicable to the underwing heat addition problem will be given.

It is observed in reference 1 that for combustion processes occurring in flowing fuel-air mixtures, effects owing to changes in chemical composition are small compared with effects owing to changes in stagnation temperature. An excellent treatment of one-dimensional flows involving changes in stagnation temperature is given in reference 2. The basic equations for the corresponding two- and three-dimensional flows are given in references 3 and 4.

In reference 5 a graphical method for solving the basic equations is employed to analyze the underwing heat addition problem. In references 6 to 10 the basic relations are linearized and applied to aircraft external combustion problems. The results of the linearized analyses indicate a need for a study of stronger heating effects than those for which the linear theory is valid.

BASIC FLOW EQUATIONS

The basic equations of reference 4 are as follows:

The continuity equation

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \vec{v} = 0 \tag{1}$$

The momentum equation

$$\rho \frac{D\vec{v}}{Dt} + \operatorname{grad} p = 0 \tag{2}$$

The energy equation

$$\rho \frac{Dh}{Dt} - \frac{Dp}{Dt} = Q_V \tag{3}$$

The equations of state

$$\left.\begin{aligned} h &= h(s,p) \\ \rho &= \rho(s,p) \\ s &= s(p,\rho) \end{aligned}\right\} \tag{4}$$

The continuity and momentum equations are identical to the usual inviscid relations for adiabatic flow. If the heat addition were not assumed reversible, to be consistent it would be necessary to add terms depending on a bulk viscosity parameter in the momentum equations. However, we will assume reversible heating and will briefly investigate the applicability of this approximation to real flows.

The energy equation is the usual relation for inviscid flow except for the addition of the term on the right, Q_V, which is the distribution of heat addition per unit volume per unit time. In reference 4, the quantity Q_V is replaced by ρq in which q is the heat power per unit mass rather than per unit volume.

The equations of state, not given explicitly here in order to preserve generality, will be specified when needed.

From these basic relations, it can be shown that in many cases the flow is irrotational, and the flow field can be expressed in terms of a velocity potential. Techniques applied to inviscid flow problems, such as linearization, the method of characteristics, etc., can be applied to two- and three-dimensional flows with reversible heat addition.

APPLICABILITY OF BASIC FLOW EQUATIONS TO REAL GAS FLOWS

A very general set of fluid flow relations is given in reference 11, chapter 11. These equations apply to any continuum, radiating, reacting gas flow when the translational degrees of freedom are nearly in equilibrium. Even turbulent flows and flows involving magnetohydrodynamic effects must be determined in part by the relations given there.

Our continuity and momentum equations follow directly from the equations of reference 11 after the deletion of viscous and body force terms. For a reacting gas mixture, the pressure appearing in these equations is the total pressure exerted by the mixture, and ρ is the total density. However, it is expedient to use an approximation wherein the quantity p appearing in our equations is taken to be the partial pressure of the nitrogen and that fraction of the oxygen and products of combustion corresponding to the number of oxygen molecules present, free and combined. Similarly, the density ρ appearing in our equations is taken to be that part of the true density which is contributed by the air. These two approximations amount to neglecting the fuel mass flow and forces due to the fuel mass flow, the presence of the fuel being taken into account only in the energy equation. For combustion in rocket motors, the forces due to fuel mass flow cannot be neglected because they are the largest forces present. But in air-breathing combustion systems, these forces are minor at vehicle velocities up to about half of satellite velocity, depending on the fuel.

Our energy equation can be related to equation (11.1-4) of reference 12, which, with the deletion of terms associated with heat conduction, diffusion processes, viscous dissipation, and body forces, is the expression

$$\frac{D\hat{U}}{Dt} = -\frac{1}{\rho}\,\mathrm{div}\,\vec{q}_R - \frac{p}{\rho}\,\mathrm{div}\,\vec{v} \tag{5}$$

where $\hat{U}$ is the internal energy per unit mass of the reacting mixture and $\vec{q}_R$ is the energy flux vector due to radiation.

The quantity $\operatorname{div} \vec{q}_R$ is equal to $(R - A)$ where R is the radiation energy emitted by the gas per unit volume per unit time and A is the corresponding absorption of radiant energy. Equation (5) can then be written

$$\frac{D\hat{U}}{Dt} = \frac{A - R}{\rho} - \frac{p}{\rho} \operatorname{div} \vec{v} \tag{6}$$

In the actual process of absorption of electromagnetic or nuclear radiation (α particles, β particles, fission fragments, etc.), the gas atoms are excited at levels of energy which are large compared to ordinary thermal energies. Strictly, the excited atoms should be treated as separate chemical species and rate constants found for the transitions to lower energies by collision and by radiation. However, if these rates are sufficiently fast or if the fraction of atoms excited is small, such details are unimportant and only the net absorption $(A - R)$ need be considered.

Using the continuity equation and the identity

$$\hat{U} + \frac{p}{\rho} = h + \hat{U} + \frac{p}{\rho} - h$$

one can express equation (6) as

$$\frac{Dh}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = \frac{A - R}{\rho} - \frac{D}{Dt}\left[\hat{U} - \left(h - \frac{p}{\rho}\right)\right] \tag{7}$$

This relation follows from the previous one for an arbitrary definition of the quantity h. Several definitions are of interest. One such is to take h to be that part of the enthalpy which is in equilibrium so that h is a function of the temperature and pressure. Then the quantity $[\hat{U} - (h - p/\rho)]$ is a part of the internal energy which is not a function of the temperature and pressure alone, but instead depends on the temperature and pressure history through reaction-rate relations given in references 11, 12, and elsewhere.

For the sake of definiteness, h is here defined to be the same function of entropy and pressure as the enthalpy of air in equilibrium.

Our energy equation follows from equation (7) by setting the right side of equation (7) equal to Q_V/ρ, that is, by imposing the relation

$$\frac{Q_V}{\rho} = \frac{A - R}{\rho} - \frac{D}{Dt}\left[\hat{U} - \left(h - \frac{p}{\rho}\right)\right] \tag{8}$$

In accord with the previous approximations regarding pressure and density, the quantity $h - p/\rho$ is the internal energy of air at equilibrium at the existing temperature and pressure. Thus the quantity $[\hat{U} - (h - p/\rho)]$ is the difference between the total internal energy of the reacting mixture and the internal energy of air at the existing temperature and pressure.

As previously mentioned, our momentum equations are consistent with the assumption of reversible heat addition. This means that the contribution of the fuel to the entropy is neglected as well as its contribution to the pressure and density. In other words, the entropy is taken to be the same function of pressure and density as is the entropy of air in equilibrium.

Equation (8) must ultimately be taken into account in a complete theory of heat addition, but for the present we join earlier investigators in seeking the consequences of taking the quantity Q_V to be an independent variable. The resulting solutions may be useful as the first step in a more exact iterative solution of combustion problems, nonequilibrium flow problems, and flow problems in which radiative heat transfer is taken into account.

It should be emphasized that the theory of reversible heat addition defined by equations (1) to (4), although only approximately related to real gas flows, is a self-consistent theory which can be analyzed rigorously.

FLOW WITH SPECIFIED THERMODYNAMIC PROCESS

The present study was undertaken to provide a basis for analysis of the underwing heat addition problem. The distribution of heat addition can be specified at the outset. Numerical examples of the results for this case will be presented. However, we will first consider an alternative procedure of specifying the thermodynamic process in the entire flow field and then solve for the required distribution of heat addition. It will be seen that this alternative leads to simplification because it is possible to solve for the pressure, density, and velocity fields independently of the energy equation in this case.

If a thermodynamic process is specified, the pressure is a known function of density (i.e., the gas is barotropic). In that case $(1/\rho)dp$ is a perfect differential, which can be expressed by the relation

$$\frac{1}{\rho}\,dp = -d\left(w - \frac{p}{\rho}\right) \qquad (9)$$

where w is the work done per unit mass by the element of gas under consideration; dw is a perfect differential only if a thermodynamic process is specified as is here the case. If dw is a perfect differential, it can be shown that the flow remains irrotational when it originates in an irrotational region and is continuous, whether steady or unsteady.

For irrotational flow, the momentum equations can be written in terms of a velocity potential φ and integrated to obtain the relation

$$w - \frac{p}{\rho} = \varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right) - \frac{p_o}{\rho_o} \tag{10}$$

Use of this relation together with the continuity equation leads to the equation

$$\varphi_{xx} + \varphi_{yy} = \frac{d\rho}{dp}\left\{\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_t + \varphi_x\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_x + \varphi_y\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_y\right\} \tag{11}$$

The quantity $d\rho/dp$ is a function of $(w - p/\rho)$ which is determined by the equations of state and the specified thermodynamic process. It follows from equation (10) that $d\rho/dp$ is a known function of $\varphi_t + (1/2)(\varphi_x^2 + \varphi_y^2)$.

As an example, consider a thermodynamic process specified by the relation for a polytropic gas

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^g \tag{12}$$

In this case the equations of state are not needed to arrive at the relation

$$\varphi_{xx} + \varphi_{yy} = \frac{\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_t + \varphi_x\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_x + \varphi_y\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]_y}{g\frac{p_o}{\rho_o} - (g-1)\left[\varphi_t + \frac{1}{2}\left(\varphi_x^2 + \varphi_y^2\right)\right]} \tag{13}$$

This differential equation is identical to the potential equation for isentropic flow of an ideal gas, except that the ratio of specific heats is here replaced by the constant g, which is not necessarily a ratio of specific heats. Yet no use has been made of explicit equations of state in the derivation. Consequently this result applies to any gas in equilibrium. The energy equation, not previously used, can be used together with the equations of state to find the distribution of heat addition Q_v required to produce the flow. For an ideal gas, this relationship is given by the expression

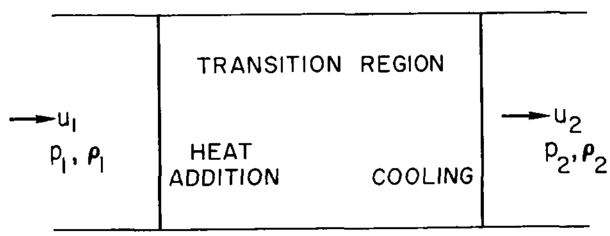
$$Q_v = \left(\frac{\gamma - g}{\gamma - 1}\right) p_o \left\{1 - \left(\frac{g-1}{g}\right) \frac{\rho_o}{p_o} \left[\varphi_t + \frac{1}{2} (\varphi_x{}^2 + \varphi_y{}^2)\right]\right\}^{g/g-1} (\varphi_{xx} + \varphi_{yy}) \quad (14)$$

Although gases in equilibrium cannot have values of the ratio of specific heats less than 1, nor greater than 5/3, flows corresponding to values of g from minus infinity to plus infinity can be constructed, if it is possible to supply the required distribution of reversible heat addition to the flow without other effects.

DISCONTINUITIES

We will now return to the solutions of the basic equations with the distribution of heat addition specified at the outset. Most arbitrary distributions of heat addition lead to rotational flows and are difficult to treat analytically. However, classes of irrotational analytical solutions can be found which are applicable to the underwing heat addition problem.

Consider a stream tube passing through a normal shock wave as depicted in sketch (a). Ahead of and behind the wave the flow is uniform with a transition region between. No knowledge of conditions in the transition region is required to obtain the shock-wave relations. However, it is interesting that the transition can be achieved by reversible heating and cooling; that is by heating in the forward part of the transition region until the Mach number reaches 1, and cooling in the rear part of the transition region until the amount of heat energy extracted is equal to the amount previously added. Since heat is extracted at a higher temperature than it is added, there is a net entropy increase equal to that which occurs in a shock wave.



Sketch (a)

We wish to consider other discontinuities in which there is a net addition of heat energy. In that case, for an ideal gas, the counterpart of the Rankine-Hugoniot relation is the expression

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma - 1) + (\gamma + 1)\frac{p_2}{p_1}}{(\gamma + 1) + (\gamma - 1)\frac{p_2}{p_1} + (\gamma - 1)\frac{2\mu}{p_1/\rho_1}} \tag{15}$$

where μ is the total enthalpy increase across the discontinuity. For present purposes, μ is taken to be an independent variable. In the special case where the value of μ is such that the Mach number behind the discontinuity is equal to 1, the Chapman-Jouguet rule is satisfied, and the discontinuity is a detonation wave. In practice, other values of μ can be achieved by combustion alongside of struts inserted into the transition region, or by combustion of fuels with high reaction rates at the ambient temperature. We assume that other dimensions are large compared with the thickness of the transition region in treating the transition as a discontinuity. When μ is not equal to zero, jumps from supersonic to any lower Mach number down to that corresponding to a shock wave and from subsonic to any higher subsonic Mach number are possible, without the necessity of heat transfer from cold to hot regions.

Oblique and curved discontinuities with heat addition can be constructed from the results for the normal case in analogy to such constructions for shock waves. In general, curved discontinuities introduce rotation in the flow.

The pressure jump is given by the relation

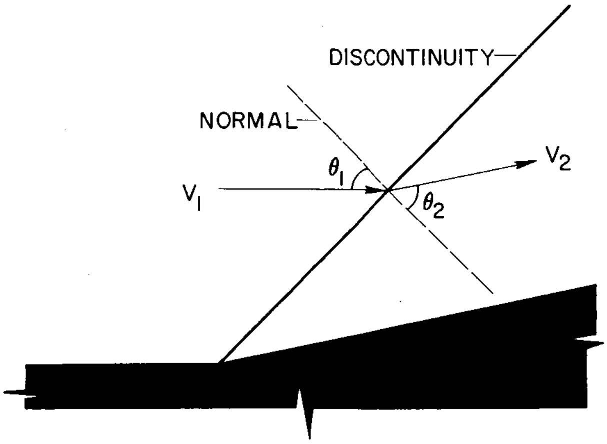
$$\frac{p_2}{p_1} = \frac{1 + \gamma M_{n_1}{}^2}{1 + \gamma M_{n_2}{}^2} \tag{16}$$

where M_{n_1} is the incident Mach number normal to the discontinuity and M_{n_2} is the normal Mach number behind. The total enthalpy increase μ is given in terms of initial conditions and normal Mach numbers by the relation

$$\frac{\mu}{p_1/\rho_1} = \frac{\gamma}{\gamma - 1}\,\frac{(M_{n_1}{}^2 - M_{n_2}{}^2)[\gamma M_{n_1}{}^2 M_{n_2}{}^2 - 1 - \frac{1}{2}(\gamma - 1)(M_{n_1}{}^2 + M_{n_2}{}^2)]}{M_{n_1}{}^2(1 + \gamma M_{n_2}{}^2)} \tag{17}$$

Equations (15), (16), and (17) apply to normal or oblique discontinuities. Sketch (b) depicts a convenient set of angle coordinates for the oblique case. The following expressions relate these angles to the independent variables, M_1, M_{n_1}, and M_{n_2}:

$$\cos\theta_1 = \frac{M_{n_1}}{M_1} \tag{18}$$



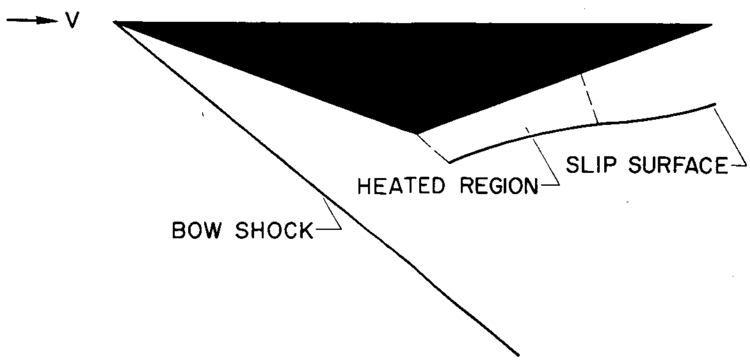
Sketch (b)

$$\cos \theta_2 = \frac{M_{n_2}}{M_2} \tag{19}$$

$$\tan \theta_2 = \frac{M_{n_1}{}^2(1 + \gamma M_{n_2}{}^2)}{M_{n_2}{}^2(1 + \gamma M_{n_1}{}^2)} \tan \theta_1 \tag{20}$$

UNDERWING HEAT ADDITION PROBLEM

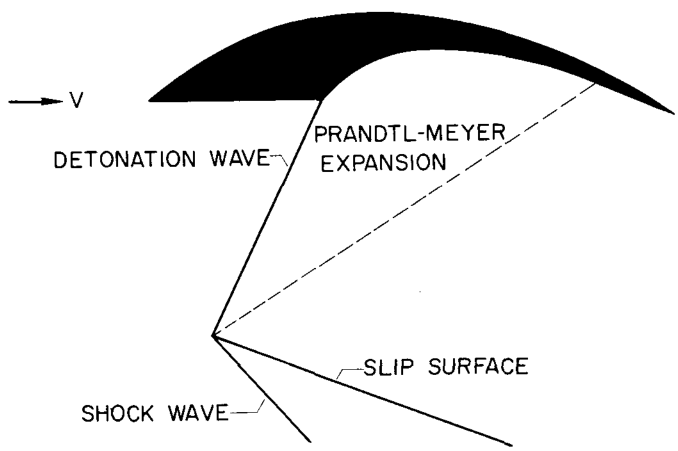
An important application of the reversible heat addition theory is in the evaluation of underwing heat addition or other aircraft external combustion schemes. This idea has been treated in references 5 to 10, and elsewhere. Sketch (c) is a sketch of a two-dimensional wing with heat



Sketch (c)

addition in the region below the wing. In the linearized approximation the flow is irrotational, and it is found that for given heating there are optimum values of wing thickness ratio and angle of attack. Also, according to linear theory, the amount of heating should be as large as possible or zero depending on the flight Mach number and engine characteristics. It turns out that the interesting values of heat addition are beyond the range of validity of the linear theory.

In the exact problem, the flow is rotational, and numerical methods are required for solution. A series of exact numerical solutions for heating under a flat plate have been obtained in reference 13. In the cases considered, the results indicate that the linear theory overestimates the aircraft performance obtainable from underwing heat addition. In reference 10, an extension of the linear theory is used to show that the performance would be improved by disposing the heat along forward-going characteristics under the wing. In an attempt to verify this prediction using exact solutions, several possibilities for using the techniques previously discussed in this work are open. Sketch (d) depicts



Sketch (d)

an airfoil shaped so as to maintain irrotational flow in the presence of heating. The heating is disposed along forward-going characteristics under the wing by means of an oblique detonation wave or other type of oblique discontinuity with heating. The detonation wave is followed by a Prandtl-Meyer expansion, which can include further heating, if a value of the polytropic exponent g less than γ is used. At the lower end of the detonation wave, a slip surface occurs between the heated and unheated air. Also a shock wave occurs below the heated region because of the deflection of the slip surface with respect to the free stream. What might be termed complete expansion is reached when the lower surface of the airfoil becomes parallel to the slip surface. If the airfoil lower surface is continued in a straight line after complete expansion, the flow behind the expansion will be uniform.

All of the results to be presented in this work are specialized to the case of oblique detonation waves with no heating in the Prandtl-Meyer expansion. Also, for present purposes, we will concentrate on the pressure forces on the lower surface while ignoring the pressure forces on the upper surface and the friction drag.

Before the results of the computations are given, further explanation of the notation is needed. For perfect combustion efficiency, an over-all engine efficiency can be written as the dimensionless ratio TV/Q, where Q is the heat power in units of foot-pounds per second, T is the thrust in pounds, and V is the vehicle velocity in feet per second. Since thrust can be developed by combustion under a wing, the ratio TV/Q for the wing is of interest for a determination of the acceleration efficiency of the wing. A similar ratio, LV/Q, takes the place of L/D in the range equation. For a conventional aircraft, the range is proportional to the product of L/D times the engine efficiency TV/Q. Since the thrust is equal to the drag in steady flight, this product becomes LV/Q.

In figure 1 plots are presented of the quantity LV/Q versus the power coefficient C_Q for flight Mach numbers from 5 to 10. The quantity C_Q is a measure of the throttle setting or, more precisely, it is the dimensionless ratio $Q/(1/2)\rho V^2 CV$. Also shown is the lift coefficient resulting from the pressure coefficient on the lower surface. The prediction of reference 10 that values of LV/Q greater than the flat-plate linear-theory value can be achieved is verified for small values of the power coefficient. However, it will be seen that for practical values of the power coefficient in the cases considered, the linear theory overestimates the performance, in agreement with the findings of reference 13. Strictly, the ratio LV/Q should be evaluated at zero total drag. However, to avoid specifying the airfoil upper surface and the friction drag, the values of LV/Q in figure 1 have been computed for zero pressure drag on the lower surface. Hence there remains a net drag of the same order as the friction drag. This will have a minor effect on range when the lift coefficient is large compared to about six times the friction drag coefficient. Consequently, the results in figure 1 for lift coefficients less than about 0.05 are not of practical significance. The results for Mach numbers 5 and 7 require heat addition less than that corresponding to the stoichiometric ratio for gasoline. In the Mach number 10 plot, the power coefficient corresponding to the stoichiometric ratio is noted.

For the results shown in figure 1, the Prandtl-Meyer expansion was extended as far as possible and a straight section of airfoil was added after that in order to develop the maximum lift consistent with zero pressure drag on the lower surface. If the airfoil is cut off at the point where the lower surface becomes horizontal, the thrust is a maximum. In figure 2 plots are presented of TV/Q versus power coefficient under this condition. The thrust coefficient and lift coefficient resulting from the pressure coefficient on the lower surface are also shown. It can be noted that engine efficiencies of 0.10 to 0.12 can be achieved by means of combustion under a wing compared with values of 0.3 or higher for conventional ram jets. However, rather large lift coefficients occur as a by-product of the underwing combustion.

CONCLUSIONS

Methods have been described for obtaining exact solutions (within the framework of the theory of reversible heat addition) applicable to the evaluation of aircraft external combustion schemes. Several such solutions have been applied to the underwing heat addition problem. The results show that the type of external combustion considered may not be advantageous in the Mach number range from 5 to 10, if conventional ram-jet engine efficiencies greater than 1/3 can be achieved. However, certain problems connected with radiant heat transfer and variable geometry may be lessened in the case of external combustion. If lower efficiencies must be accepted, the results given should be useful for estimating the performance to be expected from underwing heat-addition schemes.

SYMBOLS

A	radiation energy absorbed per unit volume per unit time
C	airfoil chord measured from detonation wave
C_L	lift coefficient, $\dfrac{L}{(1/2)\rho V^2 C}$
C_Q	power coefficient, $\dfrac{Q}{(1/2)\rho V^2 CV}$
C_T	thrust coefficient, $\dfrac{T}{(1/2)\rho V^2 C}$
D	drag
$\dfrac{D}{Dt}$	substantial derivative, $\dfrac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$
g	polytropic exponent
h	enthalpy of air per unit mass
L	lift
M	free-stream Mach number
M_{n_1}	Mach number normal to discontinuity on upstream side
M_{n_2}	Mach number normal to discontinuity on downstream side
p	pressure

q	distribution of heat addition per unit mass per unit time
Q	total heat energy addition per unit time (same units as LV)
Q_V	distribution of heat addition per unit volume per unit time
R	energy radiated per unit volume per unit time
t	time
T	thrust
$\hat{U}$	internal energy per unit mass
$v,\vec{v},\vec{V}$	velocity
w	work done per unit mass
x,y	Cartesian coordinates
γ	ratio of specific heats
η	engine efficiency
θ_1	angle between incident stream and normal to discontinuity
θ_2	angle between normal and stream behind discontinuity
μ	total enthalpy increase across discontinuity per unit mass
ρ	density
φ	velocity potential
φ_x,φ_y	velocity components

Subscripts

o	reference value
1	conditions ahead of discontinuity
2	conditions behind discontinuity
n	normal component
x,y,t	partial derivative with respect to subscript

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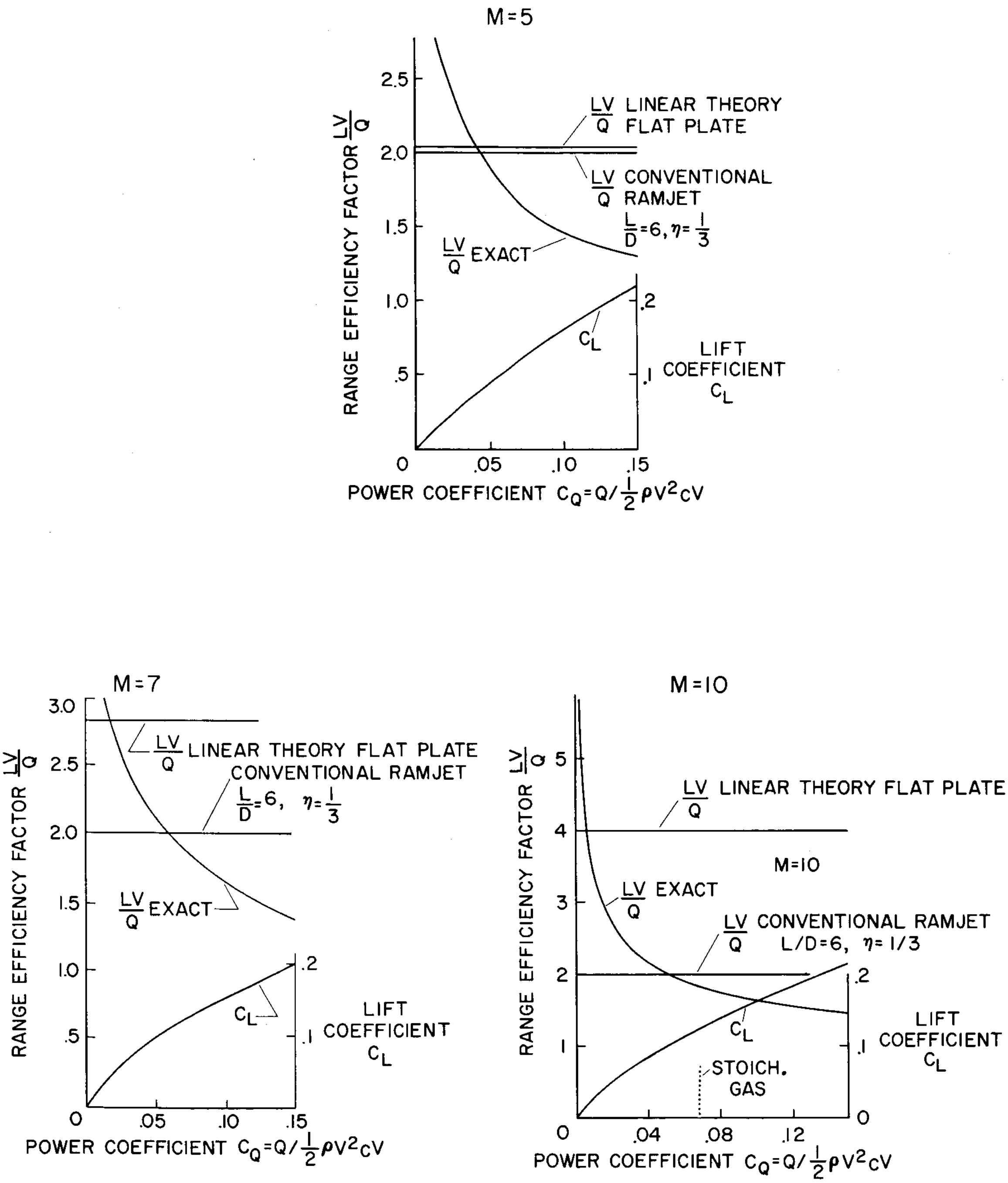

Figure 1.- Range efficiency for underwing heat addition.

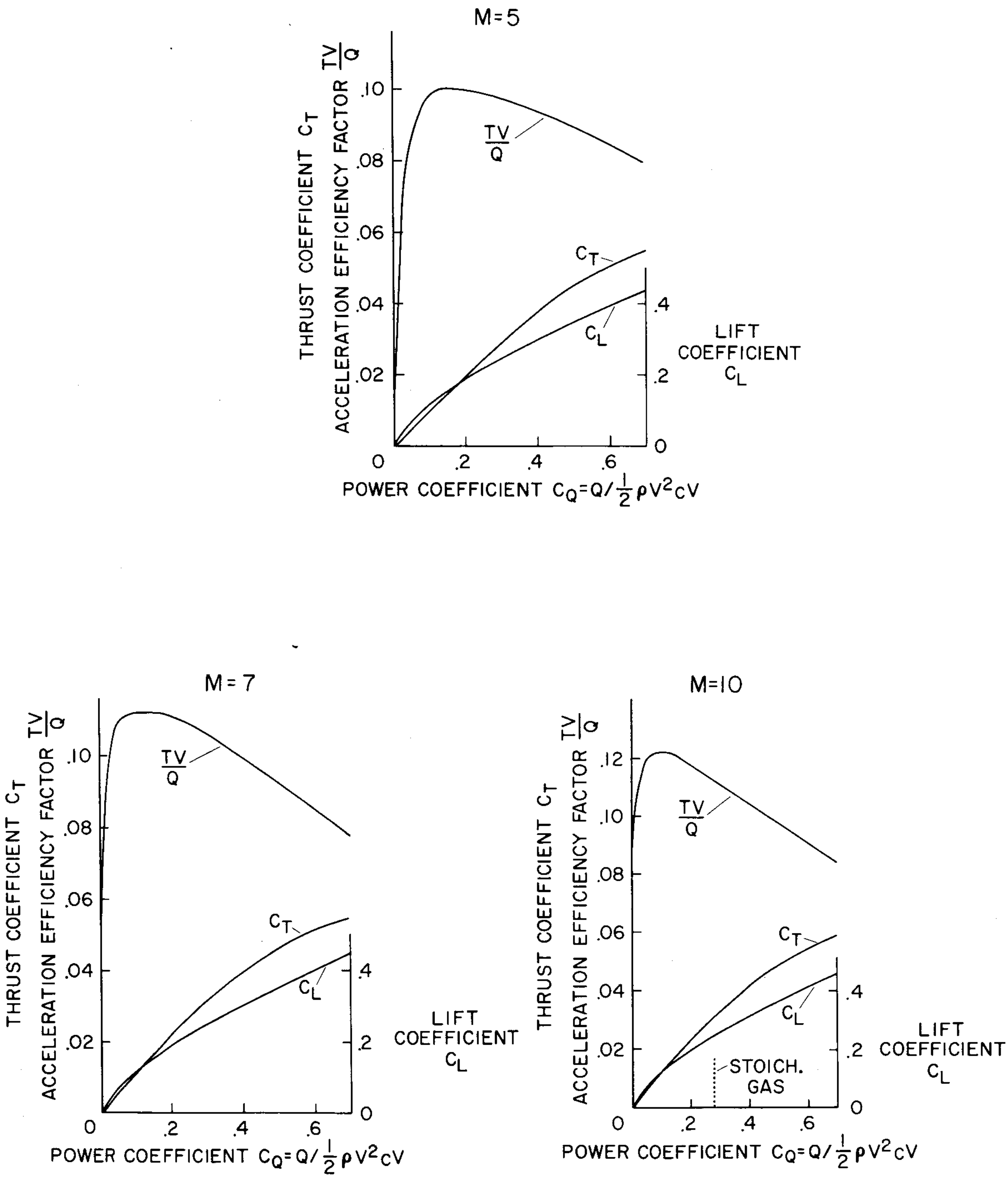

Figure 2.- Engine efficiency for combustion under airfoil.